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DEVELOPMENT OF FLUID MECHANICS AT THE LAVRENT'EV INSTITUTE OF THE SIBERIAN DIVISION OF THE RUSSIAN ACADEMY OF SCIENCES IN 1986–1996

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The studies in fluid mechanics performed over the last decade at the Lavrent'ev Institute of Hydrodynamics of the Russian Academy of Sciences were to a large degree extensions of the lines of research that were the choice of Academician M. A. Lavrent'ev, the founder of the Siberian Division. The results of the studies performed up to 1987 are presented in [1]. In this paper, we give a brief review of the advancements in the field of hydromechanics at a new stage where, along with the traditional topics that date back to M. A. Lavrent'ev, new promising lines of research have evolved and been investigated. In addition, some results of research at some other Institutes of the Siberian Division of the Russian Academy of Sciences whose lines of investigation are intimately adjacent to those of the Lavrent'ev Institute of Hydrodynamics are also touched upon in this review.

Group Analysis of the Equations of Fluid Dynamics. (1) This line of investigation, as applied to the construction of exact solutions of the equations of motion for gases and fluids, was developed in the course of realization of the SUBMODELS program, which was first presented in [2]. This program is based on the fact that many "large" mathematical models that describe physical processes in the form of a system of differential equations E have high symmetry, namely, they *admit* a fairly wide *continuous* group G of transformations for the subspace of independent and dependent variables.

The object of the SUBMODELS program is to reach the limit of the possibilities involved in such symmetry to find the class of exact solutions of the system E. Although, in the world literature, there are many examples of using symmetry properties for this purpose, the problem of reaching the limit of these properties was first posed in the SUBMODELS program.

The idea of the program is based on the fact that any subgroup $H \subset G$ is a source of exact partial solutions. The search for these solutions reduces to a *submodel* — the factor system E/H. The latter is simplified compared with E, for example, by reducing the dimension for independent variables. Therefore, if the system E admits the known basic (widest) group G, all possible subgroups of the group G will act as H. Thus, the solution of the purely algebraic problem of compiling the list of all subgroups of the given group G plays an important role in the SUBMODELS program. Actually, it suffices to list the subgroups $H \subset G$ up to the *similarity* in G, which is accomplished by the internal automorphisms of the group G. The complete list of unlike subgroups $H \subset G$ is called the *optimal system* of subgroups and denoted by ΘG.

The passage to the equations of the submodel E/H consists in establishing additional relations among the invariants of the group H that allows one to determine the desired functions and analyze the compatibility of these relations with the equations E.

A detailed description of the SUBMODELS program and the main algorithms of its realization are given in [3]. By virtue of the well-known correspondence between the Lie groups and *algebras*, the Lie algebras *of operators* L are used in the calculations, and group transformations are reconstructed, if necessary, by integration of definite systems of ordinary differential equations. In this case, the purely algebraic part of the analysis consists in constructing the optimal system of subalgebras ΘL using the algorithm described in [4].

The results of the initial stage of realization of the SUBMODELS program for the *equations of gas dynamics* (EGD) are presented in [3]. They include the *group classification* of the EGD by the *equation of*

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state for a gas $p = F(\rho, S)$, where p is the pressure, ρ is the density, and S is the entropy, and the complete list of 13 unlike *invariant* submodels with *three* independent variables for the function F of the general form. In this case, the EGD admit the 11-parameter Lie group G_{11} (or the corresponding Lie subalgebra L_{11}). The group G_{11} is an isomorphic normal extension of the classical Galilean group $R^4(t, x, y, z)$ of transformations of space and time due to homothety, i.e., the uniform extension of space R^4. The calculated optimal system of subalgebras ΘL_{11}, which consists of 220 representatives, is given in the form of a table in [3].

Each of these representatives induces, generally speaking, several different submodels of the EGD. For their classification, Ovsyannikov [5] introduces the notion of the *type* (σ, δ) of submodel, where the *rank* σ is equal to the number of invariant independent variables in the equations of the submodel, and the *defect* δ is the number of "superfluous," noninvariant desired functions. For the EGD, this number can take values $0 \leqslant \sigma \leqslant 3$ and $0 \leqslant \delta \leqslant 4$. Submodels of type $(\sigma, 0)$ are called *invariant*, and those of type (σ, δ) for $\delta > 0$ are called *partially invariant*. The latter, in turn, are subdivided into *regular* (where the invariant independent variables do not contain desired functions) and *irregular* (where desired functions are contained among the independent variables in the equations of the submodel). The complete list of possible types of submodels for the EGD (for F of the general form) is given in [6].

It should be noted that, in contrast to invariant submodels, partially invariant submodels form overdetermined systems of differential equations E/H, and the question of the existence of their solutions is not trivial. Ovsyannikov proved [7] that there are solutions for all regular submodels of type (2,1) and gave their correct description.

The realization of the SUBMODELS program for the EGD with an arbitrary F has been mainly accomplished to date. A more detailed analysis of the physical content of a number of specific submodels is given in [8–15].

For specific equations of state for a gas, the number of resulting submodels of the EGD increases by an order of magnitude. For example, in the case of a polytropic gas with $F = g(S)\rho^\gamma$ ($\gamma = \text{const}$), the admissible Lie algebra is extended to L_{13}, and the optimal system of subalgebras ΘL_{13} calculated to date consists of 1342 representatives [16]. For adiabatic exponent $\gamma = 5/3$, the admissible Lie algebra is extended to L_{14}, and ΘL_{14} has more than 2000 representatives [17].

(2) The monograph of Andreev et al. [18] deals with problems of group analysis and construction of exact solutions of the EGD (for an incompressible fluid) in Lagrangian coordinates. The transformation from Eulerian to Lagrangian coordinates is a nonlocal transformation, and, therefore, the admissible Lie group in Lagrangian coordinates was calculated independently. A physical interpretation of a number of new symmetries is given.

Wide classes of exact solutions of the Euler equations are given, which describe, as a rule, unsteady vortex motion. Representations for the solutions in Lagrangian coordinates include arbitrary functions of time and space coordinates. This makes it possible to study various initial boundary-value problems. It has been shown that Gerstner's trochoidal waves on the surface of an infinitely deep fluid are described by an invariant solution. A group explanation for the experiments of J. I. Taylor on rotating fluids is given.

(3) Shugrin [19–22] constructed the equations of two-velocity gas dynamics using the general principles of thermodynamics, the Galilean invariance, the laws of conservation of mass, energy, and momentum, and some special structural hypotheses. Central to the construction are the tensor classification of the state parameters of the system, the corresponding tensor classification of basis equations [19, 20], and the assumption of two-velocity thermodynamics [21].

The equations are constructed in two steps. The equations of "ideal two-velocity hydrodynamics," which contain only the first derivatives [21], are constructed in the first step. In the second step, based on the general Onsager principle, these equations are supplemented by "diffusion or dissipative" components containing second derivatives [22]. The approach developed in these papers provides a rational basis for the construction of equations for complex multicomponent systems, thus allowing one to obtain a consistent description.

Flows with Free Boundaries. Surface and Internal Waves. (1) A number of new theorems on the existence and uniqueness of solutions to the corresponding boundary-value problems in the classical exact formulation was proved in this line of investigation. A theory of the Conley topological index was constructed for smooth functionals defined on completely dense, open subspaces of the Hilbert space, whose linear operators generated by the second variation have regions of continuous spectrum [23]. These results were used to prove the nonuniqueness of the solution of the classical problem of a potential solitary wave on the surface of an ideal incompressible fluid.

A topological theory of disturbances for nearly symmetrical functionals was developed, and a method for obtaining the lower bound for the eigenvalues of nonlinear variational operators was proposed which is based on a combination of estimates of the index of the critical point from the Morse theory and estimates of the dimension of the negative natural subspace for the operator generated by the second variation of the functional at this point. The results obtained were used to prove the existence of periodic solutions of the nonlinear hyperbolic equation [24].

The initial boundary-value problem for the system of equations describing the dynamics of plane vortex surface waves in an eddy fluid of infinite depth has been studied [25]. It was proved that the problem is uniquely solvable locally in time for functions of finite smoothness. It has been shown that the correctness condition of this problem coincides (formally) with the correctness condition of the problem of eddy-free flows: the pressure gradient on the free surface should be directed inward in the fluid.

The problem of two-dimensional potential flow of a heavy incompressible ideal eddy-free fluid from beneath a flat horizontal shield has been solved in an exact formulation. The bottom is considered even and horizontal. The corresponding boundary-value problem contains the parameter $\lambda = gh_0U^{-2}$ (the square of the inverse Froude number), which distinguishes the supercritical ($\lambda < 1$) and subcritical ($\lambda > 1$) flows. It was proved that, for $\lambda = 1 - \delta$ and a sufficiently small $\delta > 0$, there is a nontrivial solution which coincides with accuracy to $\delta^{1/4}$ with half a solitary wave [26]. It was also proved that for $\lambda = 1 + \delta$ (for sufficiently small $\delta > 0$) there is a solution which, at infinity, behaves as a periodic wave [27].

Some features of nonlinear waves in a stratified medium do not have direct analogs in a homogeneous fluid. Among them is the existence of smooth bores, which are steady wave configurations in the form of a continuous transition which, for $x \to \pm\infty$, relates a pair of different horizontal flows. The existence of a family of exact solutions of the Euler equations that describe a bore in a two-layer fluid [28, 29] was proved. To this end, a special construction procedure for conservative problems of branching theory with nontrivial symmetry was developed [30].

Smooth bores in a two-layer fluid were realized in experiments in which undisturbed layers were at rest [31] and moved relative to one another [32].

Protopopov [33] developed a numerical algorithm for two-dimensional unsteady potential flow of an ideal fluid with a free surface and showed its effectiveness using as an example the problem of reflection of a solitary wave from a vertical wall [34]. The same author [35, 36] studied the problem of generation of soliton-type waves ahead of a moving source of fluid disturbance.

(2) Significant advances have also been made in the construction and study of approximate models that describe fluid flows in "narrow" regions.

Teshukov [37] developed a fundamentally new approach to the investigation of the mathematical model of vortex long waves which generalizes the classical model of shallow water theory. In the case of plane-parallel flow of an ideal incompressible fluid, the model equations of motion have the form

$$\begin{gathered} u_t(x,\lambda,t) + u(x,\lambda,t)u_x(x,\lambda,t) + g\int_0^1 H_x(x,\lambda',t)\,d\lambda' = 0, \\ H_t(x,\lambda,t) + (u(x,\lambda,t)H(x,\lambda,t))_x = 0. \end{gathered} \tag{1}$$

Here u is the horizontal velocity component, H is the Jacobian of the transformation from Eulerian coordinates to Lagrangian coordinates, $\lambda \in [0, 1]$ is the Lagrangian variable along the vertical, x is the Eulerian coordinate along the horizontal, t is time, and g is the free-fall acceleration. The vertical velocity component $v(x, \lambda, t)$ and the vorticity $\omega(x, \lambda, t)$ are given by the relations

$$v = F_t + uF_x, \qquad F_\lambda = H(x,\lambda,t), \qquad F(x,0,t) = 0, \qquad \omega = H^{-1}u_\lambda.$$

In the case of eddy-free flow, $u_\lambda = 0$ and $H_\lambda = 0$, and system (1) coincides with the classical model of shallow water theory. Similar systems of equations were constructed for the propagation of long waves in a barotropic fluid and in an inhomogeneous incompressible fluid.

The notions of Riemann characteristics and invariants were extended to systems of type (1) [37]. A new element of the theory compared with the classical case is the appearance of a continuous spectrum of characteristic velocities. The characteristics of system (1) are given by the equation $x'(t) = k(x,t)$. The discrete values of the characteristic velocities $k(x,t)$ are given by the equation

$$g\int_0^1 \frac{H(x,\lambda',t)\,d\lambda'}{(u(x,\lambda',t)-k)^2} = 1, \qquad k \neq u,$$

and, for the continuous spectrum, $k(\lambda) = u(x,\lambda,t)$, where $\lambda \in [0,1]$. The characteristics of the discrete spectrum correspond to surface waves, and those of the continuous spectrum correspond to internal waves. Teshukov [38] proved the local correctness of the Cauchy problem for system (1) with the initial data in the region of hyperbolicity of this system. Similar results were obtained for the model of vortex flow of a barotropic fluid [39]. The characteristic properties of long-wave equations for flows with a nonmonotonous velocity profile were analyzed by Teshukov and Sterkhova [40].

A model for the flow of an ideal incompressible fluid with a hydraulic jump, which is identified with a strong discontinuity of solutions of system (1), is proposed and studied in [41]. In contrast to the classical model of shallow water theory, the strong-discontinuity relations allow one to determine not only the mean characteristics of the flow but also the velocity profiles behind the front of the jump. A model for a hydraulic jump in the flow of a barotropic fluid with a free boundary is proposed and analyzed in [42]. The main difference from the previous model is that here hydraulic jumps that decrease the level can occur.

The theory of simple waves was extended to model (1) by Teshukov [43]. Teshukov [43] and Chesnokov [44] obtained a number of exact solutions of the equations of vortex shallow water. Elemesova [45] studied the equations of motion of a barotropic fluid. The existence of simple waves was proved, and new examples of exact solutions were obtained. The theory developed allows one to analyze the two-dimensional unsteady wave motion of a homogeneous vortex fluid with allowance for nonlinear processes.

(3) Two-layer fluid flow is the simplest case of stratified flow. However, the use of this model in a long-wave approximation involves a number of fundamental difficulties. The equations of motion are of a mixed type, and, with a sufficiently large velocity shift in homogeneous layers, the Cauchy problem becomes incorrect. The laws of conservation of mass and momentum are insufficient to obtain relations for internal hydraulic jumps.

Liapidevskii [46] realized a possible method of solution of the indicated problems. The spreading of the boundary between the layers due to mixing or generation of short waves is taken into account by using a three-layer flow scheme. In the interlayer between homogeneous layers, full laws of conservation of mass, momentum, and energy are used. This allows one to obtain a closed system that does not contain empirical constants and includes relations for internal hydraulic jumps.

The resulting equations make it possible to explain a number of flow features of miscible fluids, such as the sudden decrease in the entrainment rate with transition from the supercritical to the subcritical flow regime, the possibility of controlling the location of an internal hydraulic jump and the degree of mixing by changing downstream conditions, and the generation of short-period waves at the crest of a tidal wave. Stationary solutions and traveling waves in the sublayer with the depth of the homogeneous layer much greater than the thickness of the interlayer are analyzed. This class of solutions turns out to be very wide. It includes solitary waves or "solitons" and solutions such as "jump–wave" and a "smooth bore" [46, 47].

Using the model developed, the problem of blocking for flow of a two-layer immiscible fluid over an obstacle was solved. It was shown that, even in the supercritical flow regime, a steady subcritical flow region with intense mixing between the layers is formed ahead of the obstacle. The model also describes the formation of a mixing layer and its transition into a buoyant jet [48, 49].

The structure of long waves that arise in the shear flow of a two-layer immiscible fluid over a two-dimensional obstacle at the bottom has been studied theoretically [50, 51] and experimentally [52]. Anomalous two-layer flows of the water–kerosene system, in which an obstacle sustains the propagation of nonlinear disturbances and the flow above the obstacle is completely supercritical and a change in the obstacle height does not change the upstream flow, were discovered experimentally by R. Long (1954) and P. Byens (1984).

Incident-flow parameters for which finite amplitude disturbances either cannot propagate upstream from the obstacle or their velocity is very small was indicated. This allows one to track the flow transition to a steady regime and compare the experimental and numerical results.

(4) New results for the linear theory of generation of surface and internal waves were obtained in studies of the effect of an uneven bottom on the characteristics of generated wave motion.

The plane case of diffraction of surface waves over rectangular obstacles with schlieren zones has been investigated for a partially capped trench [53] and an underwater step [54]. The plane problem of the propagation of internal waves in an exponentially stratified fluid was solved by quadratures by Korobkin [55].

The evolution of the initial disturbance of the free surface of a homogeneous fluid in the plane case [56, 57] and three-dimensional case [58] has been studied by the method of radial approximation. In the three-dimensional problem, the solution in the vicinity of the caustic was constructed by means of the Maslov canonical operator [59].

Sturova [60] studied the effect of localized bottom roughness of small height and resonance effects in the scattering of internal waves generated by a moving body from periodic bottom roughness.

Within the framework of the linear approximation of long waves on shallow water, it has been shown that periodic ridges, shore lines, chains of islands, etc. can have wave-guiding properties [61].

Nonlinear phenomena in the generation of internal waves in a two-layer fluid by a moving and simultaneously oscillating source of disturbances have been studied analytically [62]. It has been shown that allowance for nonlinear effects reduces the problem to the Shroedinger cubic equation.

New results have been obtained in experimental studies of the generation of internal waves in a stratified fluid. Bukreev et al. [63] studied experimentally and theoretically the resonance regime occurring during simultaneous translational and vibrational motion of a cylinder in a two-layer fluid, which is accompanied by considerable wave amplification compared with the cases of purely translational or vibrational motion. It has been shown that, in the vicinity of this regime, waves propagate not only behind the cylinder but also far ahead of it, the effect being described within the framework of linear theory. Bukreev and Gavrilov [64] discovered that the motion of a body at a certain critical velocity in a stratified fluid leads to the generation of soliton-like waves ahead of the body; the waves overtake the body and go to infinity. This case of generation of disturbances ahead of the body is not described by linear theory. Bukreev et al. [65] studied internal waves that arise in a pycnocline when a body moves over an obstacle. A wide range of spectral modes generated in this case was noted.

The interaction of internal waves with a submerged body has been studied experimentally. A number of interesting effects that can be important for understanding and describing this phenomenon have been discovered. Ermanyuk [66] found doubling of the oscillation frequency for the forces acting on a body in comparison with the frequency of the incident wave. Gavrilov and Ermanyuk [67] showed that, under definite conditions, diffraction of internal waves by a body leads to excitation of higher modes, and this changes radically the wave-loading characteristics. Bukreev et al. [68] discovered the previously unknown mechanisms of memory for the motion prehistory, the possibility of body drift against waves, and the unexpected orientation of the body relative to the waves. For an elliptical cylinder with one degree of freedom (the possibility of rotation relative to the immovable horizontal axis), predominant orientation and, under certain conditions, rotation of the cylinder under the action of periodic internal waves were established [69]. Bukreev [70] found the instability of internal waves generated by a cylinder in a shear flow with a large Richardson number. This contradicts the linear theory of stability, according to which the flow under these conditions should be stable, and shows that this theory is insufficient and nonlinear analysis is required.

The hydrodynamic load acting on a body moving in a stratified fluid was determined in [71–74], where the plane problem of generation and scattering of surface and internal waves by a horizontal cylindrical body moving above a pycnocline was studied. Solving the steady problem of homogeneous flow of an infinite two-layer fluid around a circle, Khabakhpasheva [75] obtained an analytical solution in the form of a quickly converging series whose coefficients are determined from recursive relations. The solution of the diffraction problem for a circular cylinder at rest in a two-layer fluid was constructed in a similar manner [76]. Numerical calculations of the total hydrodynamic load were presented for circular and elliptical cylinders. Comparison with experimental data was performed for the problem of scattering of the internal wave of the first mode by an immovable elliptical cylinder [77].

The hydrodynamics of an airfoil in flow of a multilayer fluid was studied by Gorelov and Gorlov [78–80]. The boundary-value problem of flow around an airfoil is reduced to a system of integral equations which are not generated in the extreme case of an infinitely thin airfoil. The solution of the problem of motion of a vortex source in a multilayer fluid having an arbitrary number of homogeneous layers [81, 82] made it possible to examine a wide class of boundary-value problems of motion of an airfoil near interfaces. The calculations performed for a Joukowski symmetric profile showed that the dependence of hydrodynamic characteristics on the angle of attack and on the distance of the profile from the interface is markedly affected by the thickness of the profile.

The flow around a wing with allowance for the vortex wake behind it was studied for the first time for an exponentially stratified fluid by Tkacheva [83]. The wing was modeled by an infinitely thin plate which executed small vibrations by a given law, and the vortex wake was modeled by the line of contact discontinuity of velocity. It has been shown that the classical Joukowski formula for the lift force remains valid for a thin wing in a weakly stratified fluid. Calculations of hydrodynamic forces indicate that the dependence of the lift force of a wing on the Froude number is nonmonotonic. For larger Froude numbers, the lift force in a stratified fluid is smaller than in a homogeneous fluid, and at small Froude numbers, it is larger. The moment of forces in this case tends to zero with decrease in the Froude number.

Modeling of Phase Transitions. Phase transitions are complex processes which can be divided into several stages. The main stages are the separation of phases and coagulation. The first takes a short time, during which regions occupied by different phases are formed. In the second stage, the fine structures disappear, and the geometry of the regions occupied by different phases become simpler. In the limit, the phase interface tends to a surface with a minimum area. It follows from the aforesaid that two approaches to the modeling of these phenomena are possible. In the coagulation stage, it is natural to treat the problem as a free-boundary problem with the phase interface acting as a free boundary. In this case, under the assumption that the medium occupies region Ω, the problem is formulated as follows. It is required to determine the surface $\Gamma(t)$ which separates the subregions $\Omega^{\pm}$ occupied by different phases, and the temperature field $\vartheta(x,t)$ so that the equations

$$\vartheta_t - \Delta\vartheta = 0 \ \ (\Omega^{\pm}), \qquad \nabla\vartheta \cdot \mathbf{n} + k\vartheta = 0 \ \ (\partial\Omega)$$

are satisfied.

At the desired interface, the temperature is considered continuous, as a rule, and the following additional conditions are specified:

$$\tau V = \varepsilon \mathbf{H} - \vartheta \mathbf{n}, \qquad V = [\nabla\vartheta]\cdot\mathbf{n}.$$

Here τ is the relaxation parameter, ε is a coefficient that characterizes the surface energy of interphase interaction, $\mathbf{H}$ is the curvature vector, $\mathbf{n}$ is the normal vector to the interface, and V is the velocity of motion of the interface in the normal direction. For $\tau = \varepsilon = 0$, this problem becomes the classical Stefan problem. Meirmanov [84] ($\tau = \varepsilon = 0$) and Starovoitov [85] showed that under smooth initial conditions and with the assumption that $\partial\Omega$ does not have points of intersection with the free boundary at the initial time, this problem has a unique smooth solution for small time intervals.

The second approach is based on the study of generalized solutions. In this case, the free surface appears naturally as a surface on which the internal energy of the medium undergoes discontinuity together with derivatives of the temperature and the chemical potential. The thermodynamic system is described by the density of the free energy $F(\vartheta,\varphi)$, which is a function of temperature and the order parameter; the phase concentration can be used as the latter. In this case, the energy balance equation is the basic equation. It is supplemented by additional equations that relate temperature and the order parameters.

The most general are the equations of the gradient theory of phase transitions (the Kan–Hillard and Kan–Allen equations and phase field equations), which are widely used as mathematical models that describe phase transitions at a certain microscopic level. They contain two small parameters, the relaxation time τ and the interphase parameter ε (the characteristic length of interphase interaction). The limiting process for $\tau, \varepsilon \to 0$ leads to mathematical models that describe phase transitions at the macroscopic level. One challenge of the mathematical theory of phase transitions is to ground such limiting processes and to study the corresponding asymptotic.

The following results have been obtained in this direction. Kaliev [86] studied boundary-value problems for the equations

$$(\vartheta + l\varphi)_t - \Delta\vartheta = 0, \quad \varphi_t = \tau^{-1}(h(\vartheta) - \varphi),$$

where $h(\vartheta)$ is the Heaviside function. He proved the correctness of the basic boundary-value problems and showed that their solutions converge to the solutions of the Stefan problem. Plotnikov and Starovoitov [87] examined the following boundary-value problem for the quasi-stationary system of phase-field equations:

$$\Omega: \quad (\vartheta + l\varphi)_t - \Delta\vartheta = 0, \qquad -\varepsilon\Delta\varphi + W'(\varphi) = \varepsilon\vartheta, \qquad W(s) = (s^2 - 1)^2,$$
$$\partial\Omega: \quad \varphi_n = 0, \qquad \vartheta = 0.$$

At initial time, the distribution of the internal energy was assumed to be specified. This problem was shown to have a solution which, with the parameter ε tending to zero, converges to the solution of the Stefan capillary problem with a zero relaxation parameter value.

The resulting mathematical model essentially depends on the choice of regularization. The singular limits of solutions of viscous diffusion equations and the Kan–Hillard equations have been studied to elucidate this dependence. The viscous diffusion equation is of the form

$$u_t - \varepsilon\Delta W'(u) = \varepsilon\Delta u_t,$$

where the function $W'(u)$, the derivative of the thermodynamic potential, is an N-shaped curve.

Plotnikov [88, 89] proved that as the small parameter tends to zero, the chemical potential $W'(u)$ converges strongly to a function v, and the weak limits of functions of the form $G(u)$ admit the representation $\lambda_1 G(s_1) + \lambda_2 G(s_2) + \lambda_3 G(s_3)$, in which the functions $s_i(v)$ satisfy the equation $W'(s_i(v)) = v$. Here λ_i are the phase concentrations ($\lambda_1 + \lambda_2 + \lambda_3 = 1,\ \lambda_i \geqslant 0$).

The model in which the concentration of the unstable phase is equal to zero ($\lambda_2 = 0$) describes the phase transition processes with allowance for the hysteresis effect. For this model, a qualitative study of solutions has been performed. Plotnikov [90] established that the model satisfies the irreversibility principle: if the phases are separated at the initial instant, they remain separated at the subsequent instants of time. If surface forces are the leading factor, the phase separation in the isothermal approximation is described by the Kan–Hillard equations

$$u_t(x,t) = v_{xx}(x,t), \qquad -\varepsilon^2 u_{xx}(x,t) + W'(u(x,t)) = v(x,t), \qquad x, t \in Q.$$

In this case, the solutions oscillate with period ε, and the problem consists in determining the corresponding probability distribution function. Plotnikov [91] established that in this case the slow variables — the chemical potential v and the adiabatic invariant $I = -(\varepsilon/2)\,|u_x|^2 + W(u) - vu$ — strongly converge, and the function of distribution of the limiting concentration values u is expressed explicitly in terms of the limiting values of the slow variables.

Within the framework of the classical theory of phase transitions, the medium is considered immovable, as a rule. It is of interest to extend the approach based on the gradient theory of phase transitions to problems of motion of continuous media. In this direction, for the problem of motion of a two-component viscous fluid, Starovoitov [92] developed a model of a phase field which takes into account both the capillary interaction of fluids and their mutual diffusion. The correctness of the basic boundary-value problems for this model was established, and asymptotic analysis showed that as the small parameter tends to zero, the solutions converge to the solution of the free boundary problem for the Navier–Stokes equations.

Thermocapillary Flows and Their Stability. When a nonuniformly heated fluid has a free boundary and is in a state close to zero gravity, its motion is strongly affected by the temperature dependence of the coefficient of surface tension and by the associated thermocapillary effect. The intensity of thermocapillary motion is characterized by the Marangoni number $M = æAl/\rho\nu\chi$, where $æ$ is the constant temperature coefficient, A is the characteristic temperature gradient along the free surface, l is the characteristic dimension of the flow region, and ρ, ν, χ are the density, kinematic viscosity, and thermal diffusivity of the fluid, respectively. In studies of thermocapillary phenomena, the role of this number is as important as the role of the Reynolds number in classical hydrodynamics. In real situations, the parameter M varies widely: from a quantity of the order of unity in the processes of thermocapillary drift of microbubbles to 10^4 in experiments on directed crystallization of semiconducting materials under zero gravity conditions. The Prandtl number $\mathrm{Pr} = \nu/\chi$ and the Weber number $\mathrm{We} = \sigma_0 l/\rho\nu\chi$ are also important. Here σ_0 is the coefficient of surface tension.

Andreev and Admaev [93, 94] studied the effect of these parameters on the development of axisymmetric flow in the problem of production of single crystals by zone melting. In this case, the temperature dependence of surface tension can be linear $[\sigma = \sigma_0 - æ(\theta - \theta_0)]$ or nonlinear $[\sigma = \sigma_0 + æ_1(\theta - \theta_0)^2$ is the anomalous thermocapillary effect]. In the first case, the velocity and temperature fields and the free boundary of the melt oscillate, and, in the second case, they either increase monotonically or decay monotonically with time. The corresponding steady flows have a more complex structure. Thus, for example, for $\mathrm{Pr} = 4.1$ and $M = 10^2$, two different regimes are possible in the first case, and for $\mathrm{Pr} = 10$ and $M = 0.3$ the problem can have five different solutions for the anomalous thermocapillary effect.

The stability of equilibrium positions and of steady flows occurring under the action of thermocapillary forces have been studied [95–99]. It has been shown analytically and numerically that, in the case of monotonic disturbances, allowing for the deformation of the free surface ($\mathrm{We} \neq 0$) leads not only to a considerable decrease in stability in the region of small wave numbers, but also to the appearance of a discontinuity point in the neutral curve [95, 96]. This phenomenon was explained by solution of the full problem. The neutral curve was shown to consist of two branches, each of which corresponds to its own type of disturbances. At small wave numbers, the capillary mode, which is responsible for the disturbance of the free boundary, dominates, and at large wave numbers, the thermal mode, which is related to the nonuniform heating of the liquid, dominates.

Oscillatory thermocapillary instability in a flat layer heated from below was first discovered in the classical Pearson problem [97]. It results from both the deformability of the free surface (in the region of short waves) and the interaction of capillary and thermocapillary forces at large Marangoni numbers, when oscillating disturbances of a new type arise.

It has been shown that the instability of the thermocapillary flow (in a cylindrical layer) occurring upon heating of a melting zone by external sources of heat can be caused by deformation of the surface (the capillary mode is responsible for this), by motion of the melt (the hydrodynamic mode), and by nonuniform heating of the fluid (the thermal mode) [98, 99].

The possibility of suppressing the Rayleigh–Taylor gravitational instability by the action of thermal interphase effects was shown by Badratinova et al. [100–102], who studied the stability of the equilibrium of a vapor or gas layer separating a viscous incompressible fluid from a heated solid boundary. Here the fundamental differences from the previous studies were the allowance for the viscosity of the lower lighter phase and the assumption that the thickness of this phase is finite. At the liquid–gas interface, the thermocapillary effect was taken into account, and the phase transition at the "liquid–vapor" interface was considered. At sufficiently small thickness of the vapor layer, the phase transition at the interface is the determining factor that affects the stability of equilibrium in the "liquid over vapor" system, and the thermocapillary mechanism is the dominant mechanism for the instability of the one-dimensional heat transfer from the phase to the equilibrium "liquid over vapor" system.

The conditions of suppression of the Rayleigh–Taylor instability were formulated as new similarity criteria. The results for the critical heat flow agree with the previous statement of Kutateladze that the onset of burnout is caused by the loss of stability of two-phase wall flow [102].

A phenomenological model describing the motion of an emulsion or a gas–liquid mixture under the action of thermocapillary forces and microaccelerations was formulated by Pukhnachov and Voinov [103]. The concentration of the dispersed phase is assumed small, and this allows one to obtain a closed system of equations, without empirical parameters, for the concentration, the velocity vectors of the carrier and dispersed phases, the pressure of the carrier phase, and the total temperature of the mixture.

One-dimensional motion regimes for such a system have been studied analytically and numerically, and the structure of discontinuous solutions has been analyzed. Necessary conditions of stability for a flat "emulsion–pure liquid" interface were obtained. The stability of the spatially uniform state of mixtures which describes motions at constant concentrations and constant velocities of the phases was investigated by Pukhnachov and Voinov [104]. It has been shown that one-dimensional disturbances are the most dangerous. A stability condition was obtained, from which it follows, in particular, that, in complete zero gravity, the uniform motion of lead droplets in melted aluminum is stable, and the spatially uniform state of gas–liquid mixtures at a constant temperature gradient is unstable and has a tendency to the formation of layers with a high gas content, as shown by numerical analysis.

Ryabitskii [105, 106] studied the effect of surfactants on the occurrence of thermocapillary convection. He showed that the presence of an insoluble surfactant does not stabilize equilibrium, as expected previously, but, on the contrary, leads to the occurrence of flow even at very small temperature gradients.

Antanovskii [107] explained and modeled the anomalously slow thinning of a vertical liquid film in the gravity field in the presence of a surfactant. It has been shown that because of the entrainment of the surfactant in the gravity field, a surface-tension gradient arises, which causes upward-directed Marangoni flow. This process stabilizes the film and increases the time of its existence by three orders of magnitude compared with the case of a pure liquid.

Microconvection in Liquids. Analysis of the assumptions made in the derivation of the Oberbeck–Boussinesq equations from the exact equations of motion of a viscous heat-conducting liquid indicates that the classical model is inapplicable in the case of fulfillment of the inequality $gl^3\chi/\nu < 1$, where g is the acceleration of gravity, l is the characteristic linear dimension, ν is the kinematics viscosity coefficient, and χ is the thermal diffusivity of the fluid [108]. In addition, this model, in which the velocity field is considered solenoidal, is not able to describe correctly convective flows in distinctly unsteady conditions.

In the new model of convection of an isothermally incompressible fluid proposed by Pukhnachov [108], the velocity field is no longer solenoidal. In this case, the continuity and momentum equations are exactly satisfied, and the energy equation is asymptotically satisfied.

Pukhnachov [109] proved the unique solvability of the three-dimensional unsteady problem of microconvection in a bounded region with a heat flux specified at the boundary of the region. It has been established that the classical solution of this problem is analytic for the Boussinesq parameter βT for small values of the latter (β is the volume expansion coefficient and T is the characteristic temperature gradient). The solvability of the steady problem was proved with specification of both the heat flux and temperature at the boundary of the region if the parameter $\varepsilon = \beta T$ is small. It has been shown that, in the steady case, the difference between the dimensionless velocity vectors determined using the classical model and the new model is of the order of ε [110].

Group analysis of microconvection equations in a two-dimensional unsteady case has been performed [18]. A number of invariant solutions of the indicated system have been studied, in particular, the solution describing convective flows in a vertical layer under the action of a periodic time-dependent heat flux at its boundaries [111]. The liquid-particle trajectories calculated using the two models was found to exhibit significantly different qualitative behavior at large times. This was supported by numerical studies of microconvection regimes in circular regions for liquids such as glycerin and melts of silicon and glass [112, 113]. It has been shown, in particular, that, for unsteady flows without free boundaries, the velocities calculated by the new model can be three times higher than those predicted by the traditional model.

Motion of Bodies in a Vibrating Liquid. The behavior of bodies in a vibrating liquid has been studied since the last century. It has been established experimentally that vibrations of liquids can affect greatly the motion of inclusions, cause unusual (paradoxical) phenomena, and serve as a means for controlling the inclusions. Until recently, these phenomena were explained at a qualitative, estimation, or macroscopic level, at which the mixture of a liquid and inclusions was treated as a continuous media.

A new approach to this problem was proposed in [114–118]. This approach is based on accurate formulations of problems and experiments for an individual inclusion, and it can be considered a microscopic approach in contrast to the macroscopic one. Sennitskii [114] found that a cylinder near the wall of a vessel with a vibrating fluid experiences a force attracting it to the wall. This circumstance provides an explanation for the experimentally observed behavior of bodies in a vibrating fluid in the presence of the force of gravity, i.e., the floating of bodies with a density exceeding the fluid density, and vice versa. Lugovtsov and Sennitskii [115] showed that the occurrence of this attraction force is related to the dependence of the attached mass of the body on its location in the vessel. The equations describing the motion of a sphere in a closed vessel which executes specified translational vibrations have the form

$$(m\delta_{ij} + \mu_{ij})\ddot{\xi}_j = (\rho V - m)w_i,$$

$$\frac{d}{dt}(m\delta_{ij} + \mu_{ij})\dot{X}_j - \frac{1}{2}\frac{\partial \mu_{jk}}{\partial X_i}\dot{X}_j\dot{X}_k = (m - \rho V)g_i + \frac{1}{2}\frac{\partial \mu_{jk}}{\partial X_i}\overline{\dot{\xi}_j\dot{\xi}_k}, \qquad i = 1, 2, 3,$$

where m is the mass of the sphere, V is its volume, $\mu_{ij} = \mu_{ij}(X_k)$ is the tensor of the attached mass, which depends on the location of the sphere in the vessel and on the vessel shape, ξ_i is the quickly time-varying, small deflection from the average trajectory of the sphere, and w_i is the acceleration of the vessel; the bar denotes averaging over time. The last term on the right side of the second equation results from averaging over high-frequency vibrations of exact equations and describes the indicated force.

The above circumstance, however, is not the only one that can lead to the effects observed in vibrating liquids. It is important that inclusions upon vibratory actions perform motion in different directions under nonuniform conditions. Sennitskii [116] showed that the motion of inclusions can be controlled if these inclusions are compressible bodies (a gas bubble or a compressible solid body). The predominantly unidirectional motion of inclusions is achieved by choosing an appropriate phase difference between the vibrations of the closed vessel and the periodic pressure changes in it. This theoretical result was confirmed experimentally [117, 118].

To go over from the microscopic to the macroscopic description, it is necessary, at least, to study the interaction of inclusions. As a first step in this direction, Sennitskii [119] studied the motion of a sphere caused by vibrations of another sphere. He found that the mean motion of the inclusion (the free sphere) is directed toward the periodically vibrating sphere if the density of the inclusion is higher than the fluid density, and if the ratio of densities is inverse, the direction of motion of the inclusion is reversed. This effect can be treated as the "generation" of a "gravity field" by the vibrating sphere. In this field, the free sphere "floats up" or "sinks," depending on the ratio between inclusion and fluid densities.

An important point in studies of the motion of inclusions in a vibrating liquid was the division of liquid vibrations into uniform and nonuniform [120]. If, in the absence of inclusions in a vibrating liquid, all liquid particles move at the same velocity, the vibrations are uniform. If this is not the case, the vibrations are nonuniform. Depending on the type of vibrations, qualitatively different mean motion of inclusions occurs. In particular, in the case of uniform vibrations, a solid inclusion whose density is equal to the liquid density moves together with the liquid.

Lavrent'eva [121] showed that, in the case of nonuniform vibrations generated by a point pulsating source, there is mean motion of a free sphere having the same density as the liquid. Interesting details of this motion with an arbitrary ratio of densities have been established. It has been found that the sphere approaches the source if its density is not lower than the liquid density or if its center is not too distant from the source at initial time. The sphere moves away from the source if its density is lower than the liquid density or if it is far from the source at initial time.

Filtration Flows. New mathematical models of filtration theory have been developed. Kashevarov [122] proposed a model and iteration algorithm for the transfer of pollutants by interacting flows of surface, soil, and ground water for large-scaled objects. For individual components of the complex system of nonlinear equations of interrelation, the uniqueness and existence (locally in time) of solutions of the corresponding initial boundary-value problems was proved. In particular, Antontsev and Kashevarov [123] examined the localization of solutions of nonlinear parabolic equations degenerating on the surface. A number of exact solutions of multiparameter problems for the important class of plane filtration flows with a free surface in the presence of singular points was obtained and studied by Emikh [124].

For the solution of problems of geotechnology and ecology, Pen'kovskii and Rybakova proposed models of interrelation of hydraulic and filtration processes occurring in drilling (the bore–bed system, predictions of crust formation, and location of zones of possible stalling of boring tools) [125], and also models of underground leaching [126] with allowance for unsteady filtration, convective diffusion, and mass exchange between the moving solution and the frame of the medium. Using statistical methods, Kapranov [127] developed a new approach to the study of the mechanical injection of low-concentration mixtures, such as clay solutions, into a porous medium.

Domanskii [128] and Antontsev et al. [129, 130] studied the properties of the nonlinear system of degenerating equations of two-phase filtration. These studies revealed some phenomena of theoretical and applied importance in the processes of immiscible displacement. In particular, the phenomenon of capillary entrapment of inclusions of the displaced phase both at the boundary and inside the filtration region of the liquid displacer have been established theoretically and experimentally, and criteria of destruction of such inclusions have been formulated. This provides a new insight into the nature of formation of retained oil in beds being exploited and into the development of methods of activating inflow to boreholes. Pen'kovskii [131] established recently that similar phenomena occur in the case of three-phase filtration.

Monakhov and Khusnutdinova [132] studied the conjugation of high-velocity flows of a viscous fluid in boreholes and open beds (channels) with filtration flows of the fluid in the ambient porous medium [132]. A number of variants of conjugation were examined within the framework of a boundary-layer approximation for both flows. For mutually perpendicular boundary layers in a borehole and the porous bed adjacent to the borehole, the solvability of the corresponding boundary-value problems was proved and the class of self-similar flow regimes was obtained.

Hydrodynamic Stability. Over the last decade, the region of application of the direct Lyapunov method has been considerably extended in studies of the stability of equilibrium (quiescent) states and steady flows of liquids and gases. New results have been obtained in this line of investigation. These are the exponential estimates of increase of disturbances in problems of linear instability of the quiescent states and steady flows and *a priori* estimates indicating a quadratic mean increase of disturbances in problems of linear instability of a number quiescent states. It is of interest to obtain sufficient conditions of nonlinear instability that, on the one hand, would generalize the known conditions in the sense of new definitions of stability and, on the other hand, provide information on the stability of equilibrium states and steady symmetric flows that cannot be described by the known conditions.

Vladimirov et al. [133–137], Il'in [138], and Gubarev [139, 140] give numerous examples of construction of the Lyapunov functional and inversion of the Lagrange theorem to establish the instability of quiescent states for various hydromechanical schemes against small spatial disturbances and to obtain exponential estimates.

For a stationary, axisymmetric, compressible, baroclinic vortex in the potential field of external mass forces [141] and for magnetohydrodynamic (MHD) flows of an ideal incompressible fluid with infinite conductivity [142], instability against small disturbances of the corresponding symmetry has been established and exponential estimates of increase of the disturbances have been obtained.

Sufficient conditions of nonlinear stability against spatial disturbances have been obtained for the quiescent states of an incompressible liquid with nonuniform density (continuously stratified) in the potential field of external mass forces and for a number of steady flows of compressible and incompressible fluids, including an ideally conducting fluid with a certain type of symmetry in a magnetic field with disturbances of the same symmetry [143–147].

Belov and Vladimirov [148] proved the instability against finite plane disturbances for the quiescent states of two immiscible, ideal, incompressible, capillary fluids with different densities filling a fixed cylindrical vessel in the potential field of external mass forces, and Gubarev [140] established the instability of the quiescent states of an infinite, self-gravitating, compressible medium against finite spatial disturbances [140]. For both problems, estimates were obtained that indicate a quadratic mean increase of the corresponding disturbances.

A peculiar problem of fluid flow stability is the problem of spontaneous swirling: can rotary motion occur in axisymmetric flow as a result of the loss of stability in the absence of external sources of rotation, i.e., under conditions where motion without rotation is *a priori* possible?

Gol'dshtik et al. [149–151] state that spontaneous swirling is possible. This phenomenon was called "self-rotation" or a "vortex dynamo." However, in the examples constructed there is a nonzero axial component of the angular momentum which inflows in the flow region. Lugovtsov and Gubarev [152, 153] propose a more rigorous formulation which ensures a strict control of the kinematic flow of the angular momentum and eliminates the inflow of the rotating fluid in the flow region considered. In this formulation, the question of the possibility of bifurcations of the initial axisymmetric flow as a result of the loss of stability against swirling flow (not necessarily rotationally symmetric flow) remains open.

To prove the existence of this phenomenon, it is necessary to find even one example. In an attempt to narrow the region of search for such an example, Lugovtsov and Gubarev [152–154] considered the transition of axisymmetric flow to rotationally symmetric flow and the plane analog of such transition — the occurrence of spontaneous transverse flow (perpendicular to the initial flow), which is independent of the transverse coordinate in the case of initial plane-parallel flow.

It has been shown [153] that the bifurcations axisymmetric flow–rotationally symmetric flow and the corresponding plane analog of this transition do not occur for an arbitrary compressible fluid with a variable viscosity coefficient. This result is analogous to the well-known (in "magnetic dynamo" theory) Cowling theorem on the impossibility of an axisymmetric "magnetic dynamo" and the corresponding plane analog of this phenomenon. In the case of the plane analog, this statement was shown to be also valid for a conducting fluid moving in a magnetic field, irrespective of the character of connectedness of the flow region.

The situation is different for axisymmetric flows in a magnetic field. In this case, as is shown by Lugovtsov and Gubarev [153], under definite conditions, steady swirling flows sustained by electromagnetic forces are possible, and the formulation of the problem of spontaneous swirling requires refinement. This refinement was made in [154], where it was shown that axisymmetric spontaneous swirling is impossible for a fluid with finite conductivity if the meridian section of the flow region is simply connected. In this region, the poloidal components of the magnetic field always vanish with time, and swirling becomes impossible. For a multiply connected region, this question remains open.

For an ideally conducting fluid, the character of connectedness of the flow region becomes insignificant, because, in this case, the poloidal components of the magnetic field do not vanish by virtue of freezing. Lugovtsov [155] showed that, in such MHD flows in the presence of external fields having only poloidal components, axisymmetric spontaneous swirling is possible, at least for an inviscid fluid.

As is shown in [153], rotationally symmetric flow (in the general case, on the average; averaging over the azimuthal angle) can arise only as a result of countergradient flux of the angular momentum. Although the flow considered in [149] cannot be treated as an example of spontaneous swirling, it demonstrates the possibility of occurrence of countergradient flux of angular momentum in nonaxisymmetric swirling flows. In MHD flows, the magnetic field leads to additional possibilities for the occurrence of such a mechanism, and, as is shown in [155], it is realized in axisymmetric swirling. Within the framework of the formulation proposed in [152, 153], the question of the existence of such a mechanism in nonaxisymmetric flows without magnetic field remains open.

Vortex Motions of a Fluid. A number of results obtained in studies of unsteady vortex flows based on group analysis are given in [18].

Vortex structures are of certain independent interest. In addition, they have attracted attention in connection with studies of large-scale formations in turbulent flows. Interesting results have been obtained in studies of steady vortex structures in an inviscid incompressible fluid in the presence of additional symmetries [18, 156, 157].

In the case of steady plane flows of an ideal incompressible fluid, Kaptsov [156] obtained exact solutions of the equation for the stream function $\Delta\psi = \omega(\psi)$. The form of all right sides $\omega(\psi)$ for which this equation admits generalized separation of variables is found. The solutions describe flows of the type of a source in an eddy fluid, periodic flows between two walls, motions in a rectangular cylinder, and some others.

For axisymmetric swirling flows, the equation of the stream function (in plasma physics, it is known as the Grad–Shafranov equation) with a special form of the nonlinear right side also admits separation of variables. This made it possible to obtain exact solutions corresponding to various vortex structures: an exponentially decaying vortex shielded by two walls, analogs of Taylor toroidal vortices, periodic "loop tracks," structures of the type of a "cat's eye," and some other magnetic vortex structures in a plasma [157].

In the Kirchhoff elliptic vortex, the fluid velocity is continuous and the fluid is at rest at infinity. Garipov [158] studied the spatial analog of this flow. The flow of an inviscid incompressible fluid with piecewise-constant vorticity and piecewise-constant density which undergo discontinuity on the surface of an ellipsoid was examined. In this case, a tangential discontinuity on the boundary between the ellipsoid and a linear increase in the fluid velocity at infinity are assumed. In this generalized formulation, nontrivial spatial solutions exist. All flows of such a structure have been found. It has been shown that this class of flows contains, in particular, all known generalizations of the Kirchhoff vortex (they are all plane).

Plane flows of an inviscid fluid in regions bounded by solid walls are calculated in [159–162]. A new numerical technique based on absolutely stable difference schemes was used. Effective algorithms using splitting by physical processes and Fourier discrete transform, which ensure exact satisfaction of the attachment conditions on the walls and, in the case of doubly connected regions, satisfaction of the pressure uniqueness condition, were developed to solve two-dimensional systems of the Stokes and Navier–Stokes difference equations written in the variables "stream function–vorticity."

Kuznetsov [163] examined the possibility of continuation of the Prandtl boundary layer when the pressure increases downstream. It has been established that for any pressure distribution, it is possible to indicate the initial velocity profile for which the continuation is possible if a certain inequality is satisfied which relates the quantities calculated from the data of the problem.

Linear (Tornado-Like) Vortices. The study of flows with linear vortices is important for understanding the dynamics of vortex formations in nature (cyclones, hurricanes, waterspouts, and tornadoes) and in various technical devices (centrifugal atomizers, vortex chambers, etc.). This field of investigation has been developed in both theoretical and experimental lines.

Akhmetov and Tarasov [164] report the results of an experimental study of the internal structure and evolution of the core of a tornado-like vortex occurring in the flow between two coaxial disks of the same radius, rotating at constant angular velocity in the same direction. Measurements have shown that flow of the type of a tornado-like vortex with a rigid-body rotating core is established (the average velocity field). It has been found that, with excess of a certain critical Reynolds number, the cross section of the core loses circular symmetry and acquires the shape of an oval, triangle, quadrangle, etc. Surprisingly, the number of vertices of the polygon decreases with increase in the Reynolds number, and this contradicts intuitive expectations. There is no grounding in theory to this phenomenon. With sufficiently large Reynolds numbers, the core consists of a system of smaller secondary vortices. It continuously deforms and regularly exchanges fluid with the surrounding flow by ejecting spiral sleeves, which propagate in the external flow, and entraining the external fluid as individual jets.

Makarenko and Tarasov [165] found experimentally that, in a rigid-body rotating fluid, a system of tornado-like vortices parallel to the rotation axis occurs under various disturbances. Based on this fact, they proposed a new mechanism for the occurrence of tornadoes involving the initiation of intense inertial waves in a rotating fluid [166]. It has been established that, when a cylindrical vessel with a fluid rotates at constant angular velocity and its flexible upper surface oscillates in a specified manner, a tornado-like vortex occurs in the fluid. The vorticity level of this vortex is extremely high and far exceeds the doubled angular velocity of the vessel [167]. The properties of this vortex were shown to be similar to the well-known properties of atmospheric vortices — tornadoes. This similarity provides an explanation for many facts due to the propagation of tornadoes.

Makarenko [168, 169], using a laboratory setup, showed that vortices of this type can occur under certain conditions in the interaction of a mesocyclone with roughness of the earth's surface. The conditions of occurrence are formulated in the form of criteria that allow prediction of tornadoes in nature.

The problem of the effect of rotational tangential stresses of a definite type (decaying in inverse proportion to the square of the distance from the center of rotation) on the plane free surface of a viscous fluid was solved in an exact formulation by Nikulin [170]. He showed that a linear vortex and an upward flow along it occur in the fluid, the flow being self-similar. The existence theorem was proved, and the qualitative behavior of the solution was studied. The results were used to calculate the upwelling (outside the zone of maximum winds) that occurs when a hurricane moves over the ocean.

Nikulin [171–175] used the long-wave approximation equations derived in [172], which are similar to eddy shallow water equations, to describe flows in rotationally symmetric hollow and tornado-like vortices. The steady fluid flow in the core of a vertical tornado-like vortex has been studied ignoring fluid rotation in the core [173] and taking it into account [174]. A strict criterion was obtained which expresses conditions of continuation of solutions to a finite or an infinite height. The noncontinuability of solutions is associated with vortex decay. The location of vortex decay was determined and an analytical model of this phenomenon was constructed [171]. The flow evolution in the vortex core has been studied [174]. The steady fluid flow in a hollow vortex in a tube of variable radius has been examined [175]. It has been shown that two different flow regimes are possible. A strict criterion distinguishing these two regimes is obtained. The flow was shown to be similar to ideal gas flows in tubes of variable cross section.

At present, the main flow parameters in a centrifugal atomizer are calculated using the principle of maximum discharge (PMD) proposed by G. N. Abramovich and, independently, by J. Taylor, which supplements the conservation laws which are insufficient in the general case. It follows from this principle that the flow in the atomizer nozzle must be strictly critical. Lugovtsov [176] showed that, for a centrifugal atomizer of special shape (an atomizer with a Borda nozzle), the main flow parameters are determined exactly using the conservation laws within the framework of the model of an ideal incompressible fluid. The exact results differ greatly from those obtained on the bases of PMD. Thus, for example, the nozzle flow velocity turns out to be supercritical (exceeds the critical velocity by a factor of two or more). A similar situation also arises for flows through spillways [177]. These results pose doubt on the reliability of calculations using PMD.

Turbulence. Bukreev [178] studied the transition from the laminar to the turbulent regime for the flow caused by longitudinal vibrations of a cylinder in an infinite fluid. The boundary between the laminar and turbulent regimes was determined.

Turbulent, axisymmetric, momentum-free, jet flow [179] and the vortex wake behind a sphere with compensation of the drag force [180] have been studied experimentally. A number of statistical characteristics of turbulent velocity pulsations were obtained.

The disappearance of the "memory" of the initially asymmetric location of a heat source in a symmetric turbulent mean flow has been studied experimentally and using numerical calculations [181]. It has been shown that the distributions of statistical characteristics of the temperature field tend, although slowly, to the same symmetry as the distribution of characteristics of the velocity field.

Hydroaeroelasticity. Ryabchenko [182] calculated unsteady aerodynamic characteristics for an annular blade cascade of arbitrary shape vibrating in a flow of an incompressible fluid [182] and for a blade cascade in a subsonic gas flow [183]. The limits of applicability for the assumption of plane and cylindrical sections were determined numerically.

Kurzin [184] considered the evolution of vortex wakes in the interaction of two cascades. Yudin [185] analyzed the effect of the evolution of vortex wakes on the unsteady aerodynamic characteristics of blades. An experimental and theoretical study of acoustic resonance in the aerodynamic interaction of cascades is reported in [186]. It is shown that for resonance to occur, not only must the natural frequencies and the frequencies of exciting forces coincide, but it is also necessary that a certain ratio of the number of blades in the moving cascades be satisfied.

Using the model problem of the stability of the location of a free vortex at the center of an annular cascade, Kurzin and Ovsyannikova [187] showed that, because of instability, self-excited circular inhomogeneity of the flow can occur in centrifugal turbomachines.

Kurzin et al. [188] developed a method for calculating aeroelastic vibrations of blade cascades, which brings the calculation model as near as possible to the object of investigation.

A model of an active Helmholtz-type resonator for suppressing acoustic vibrations in combustion chambers was proposed [189]. Analytical relations for determining optimal resonator parameters were obtained. This model was used to explain the sudden increase in the level of vibrations of the active part of an inductive source of energy [190].

Unsteady processes in combustion chambers have been studied in theoretical, experimental and full-scale studies. It has been shown that the hydrodynamic instability of large-scale vortex structures is responsible for excitation of low-frequency acoustic vibrations in combustion chambers [191, 192].

Water Impact. The problem of collision of liquid and solid masses belongs to the wide class of problems of unsteady fluid flow in a time-varying region whose boundary consists of a free surface, a moving solid surface, and the line of contact between them.

In the problem of impact of a solid body on a liquid, the initial stage of collision, in which the main quantities undergo significant changes, is of special interest. Just after the beginning of motion, the topology of the flow region changes: the previously absent component of the liquid boundary adjacent to the solid surface appears. Even after all possible simplifications, the problem remains linear, because the size of the region of contact is not known in advance. It is determined from the condition of restricted motion of liquid particles [193]. Within the framework of an incompressible ideal liquid model in the plane case, this condition leads to the system of two transcendent equations for the coordinate of the point of contact. In this case, the motion potential is used instead of the velocity potential, which is traditionally used in penetration problems. It was found that the form of the equation depends not only on the body geometry. This made it possible to study in detail the effect of the shape of a submerging body on hydrodynamic loads and indicate the body shapes for which these loads are extreme [194].

In the initial stage of submersion in a liquid, the velocity of broadening of the wetted area of a blunt body can exceed the local speed of sound in the liquid even when the collision velocities are not high. Therefore, ignoring the compressibility of the liquid can lead in some cases to physically unrealistic results. Korobkin [195–198] constructed an asymptotic theory of collision of a rigid body with a slightly compressible liquid. The low Mach number, which is equal to the ratio of the collision velocity to the sound velocity in the quiescent liquid, acts as a small parameter. The theory is essentially based on the ideas developed within the framework of an ideal incompressible liquid model. However, in this case, the size of the region of contact depends not only on the body shape but also on the history of body motion and fluid flow. Because of the presence of the line of contact which is not known in advance, the problem remains essentially nonlinear even after linearization of the boundary conditions and motion equations. But even this "partial" linearization is not always possible. The proposed theory allows one to describe the pressure distributions over the wetted area of the body [195, 196], the dynamics of the shock waves generated upon impact [197, 198], the formation of spraying jets [199], and cavitation phenomena [200] observed in the region of contact. Extension of the theory to the case of a deformable body is given in [201]. The majority of the results are obtained in an analytical form, and this makes it possible to analyze the above-mentioned phenomena and estimate the main characteristics. Previously, these problems have been studied only numerically.

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REFERENCES

1. B. A. Lugovtsov and L. V. Ovsyannikov, "Development of hydrodynamics at the Siberian Division, Academy of Sciences of the USSR," *Prikl. Mekh. Tekh. Fiz.*, 28, No. 4, 3–22 (1987).

2. L. V. Ovsyannikov, *The SUBMODELS Program* [in Russian], Institute of Hydrodynamics, Novosibirsk (1992).
3. L. V. Ovsyannikov, "The SUBMODELS program. Gas dynamics," *Prikl. Mat. Mekh.*, **58**, No. 4, 30–55 (1994).
4. L. V. Ovsyannikov, "Optimal systems of subalgebras," *Dokl. Ross. Akad. Nauk*, **333**, 702–704 (1993).
5. L. V. Ovsyannikov, "Regular and irregular partially invariant solutions," *Dokl. Ross. Akad. Nauk*, **343**, 156–159 (1995).
6. L. V. Ovsyannikov and A. P. Chupakhin, "Regular partially invariant submodels of gas dynamics," *Prikl. Mat. Mekh.*, **60**, No. 6, 1011–1020 (1996).
7. L. V. Ovsyannikov, "Type (2,1) regular submodels of the equations of gas dynamics," *Prikl. Mekh. Tekh. Fiz.*, **37**, No. 2, 3–13 (1996).
8. L. V. Ovsyannikov, "Isobaric motion of a gas," in: *Differ. Uravn., Minsk*, **30**, 1792–1799 (1994).
9. L. V. Ovsyannikov, "Singular vortex," *Prikl. Mekh. Tekh. Fiz.*, **36**, No. 3, 45–52 (1995).
10. S. V. Khabirov, "On the analysis of submodels of vortex motion in gas dynamics," *Usp. Mat. Nauk*, **50**, 119 (1995).
11. S. V. Khabirov, "On the analysis of invariant submodels of rank three of gas-dynamic equations," *Dokl. Ross. Akad. Nauk*, **341**, 764–766 (1994).
12. S. V. Meleshko, "Group classification of the equations of two-dimensional motions of a gas," *Prikl. Mat. Mekh.*, **58**, 56–62 (1994).
13. S. V. Meleshko, "On one class of partially invariant solutions of two-dimensional motions of a gas," *Differ. Uravn., Minsk*, **30**, 1825–1827 (1994).
14. A. A. Talyshev, "A computer program of differentiations for a finite-dimensional Lie algebra," *Vychisl. Tekhnol.*, **4**, 272–274 (1995).
15. A. P. Chupakhin, "Optimal system of subalgebras of one solvable algebra L_7," in: *Lie Groups and Their Applications*, Vol. 1, Celal Bayar Univ., Manisa, Turkey (1994), pp. 56–70.
16. S. V. Golovin, "Optimal system of subalgebras for the Lie algebra of operators admitted by the equations of gas dynamics in the case of a polytropic gas," Preprint No. 5–96, Institute of Hydrodynamics, Novosibirsk (1996).
17. A. A. Cherevko, "Optimal system of subalgebras for the Lie algebra of operators admitted by the equations of gas dynamics with the equation of state $p = f(S)\rho^{5/3}$," Preprint No. 4–96, Institute of Hydrodynamics, Novosibirsk (1996).
18. V. K. Andreev, O. V. Kaptsov, V. V. Pukhnachov, and A. A. Rodionov, *Using Group-Theoretical Methods in Hydrodynamics* [in Russian], Nauka, Novosibirsk (1994).
19. S. M. Shugrin, "Galilean systems of differential equations," *Differ. Uravn., Minsk*, **16**, No. 12, 2205–2218 (1980).
20. S. M. Shugrin, "Conservation laws, invariance, and the equation of gas dynamics," *Prikl. Mekh. Tekh. Fiz.*, **30**, No. 2, 10–18 (1989).
21. S. M. Shugrin, "Two-velocity hydrodynamics and thermodynamics," *Prikl. Mekh. Tekh. Fiz.*, **35**, No. 4, 41–59 (1994).
22. S. M. Shugrin, "Dissipative two-velocity hydrodynamics," *Prikl. Mekh. Tekh. Fiz.*, **35**, No. 4, 59–87 (1994).
23. P. I. Plotnikov, "Bifurcations of the critical points of smooth functionals and the nonuniqueness of solutions of the problem of solitary waves," *Izv. Akad. Nauk SSSR, Ser. Mat.*, **55**, No. 2, 340–366 (1990).
24. P. I. Plotnikov, "The existence of an infinite set of periodic solutions of the problem of induced oscillations for a weakly nonlinear wave equation," *Mat. Sb.*, **136(178)**, No. 4(8), 546–559 (1988).
25. V. I. Nalimov, "Unsteady vortex surface waves," *Sib. Mat. Zh.*, No. 6, 1356–1366 (1996).
26. V. I. Nalimov, "Supercritical flows from beneath a shield," *Prikl. Mekh. Tekh. Fiz.*, **30**, No. 2, 77–81 (1989).
27. V. I. Nalimov, "Subcritical flows from beneath a shield," *Prikl. Mekh. Tekh. Fiz.*, **39**, No. 1 (1998).

28. N. I. Makarenko, "Smooth bore in a two-layer fluid," *Int. Ser. Numer. Math.*, **106**, Birkhauser Verlag, Basel (1992), pp. 195–204.
29. N. I. Makarenko, "Asymptotics of asymmetrical internal waves," in: *Vychisl. Tekhnol.*, **2**, No. 4, 22–29 (1993).
30. N. I. Makarenko, "On the branching of solutions of invariant variational equations," *Dokl. Ross. Akad. Nauk*, **348**, No. 3, 302–304 (1996).
31. V. A. Ageev, V. I. Bukreev, and N. V. Gavrilov, "A new type of plane stationary waves in a two-layer liquid," *Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza*, No. 5, 187–190 (1986).
32. V. N. Gavrilov, "Smooth bores in a two-layer liquid with a velocity shear between the layers," *Prikl. Mekh. Tekh. Fiz.*, **28**, No. 3, 45–49 (1987).
33. B. E. Protopopov, "Numerical simulation of surface waves in a channel of variable depth," in: *Dynamics of Continuous Media* (Collected scientific papers) [in Russian], Novosibirsk, **84** (1988), 91–105.
34. B. E. Protopopov, "Numerical analysis of transformations of a solitary wave with reflection from a vertical wall," *Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza*, No. 5, 115–123 (1990).
35. B. E. Protopopov, "Numerical simulation of effects of generation of solitons by a moving region of surface pressure," *Prikl. Mekh. Tekh. Fiz.*, **32**, No. 3, 78–84 (1991).
36. B. E. Protopopov, "Upstream generation of solitons: numerical analysis of the dependence on key parameters," *Prikl. Mekh. Tekh. Fiz.*, **34**, No. 1, 88–94 (1993).
37. V. M. Teshukov, "On the hyperbolicity of long-wave equations," *Dokl. Akad. Nauk SSSR*, **284**, No. 3, 555–559 (1985).
38. V. M. Teshukov, "On the Cauchy problem for long-wave equations," *Int. Ser. Numer. Math.*, **106**, Birkhauser Verlag, Basel (1992), pp. 331–338.
39. V. M. Teshukov, "Long waves in an eddying barotropic liquid," *Prikl. Mekh. Tekh. Fiz.*, **35**, No. 6, 17–26 (1994).
40. V. M. Teshukov and M. M. Sterkhova, "Characteristic properties of the system of equations of shear flow with a nonmonotonic velocity profile," *Prikl. Mekh. Tekh. Fiz.*, **36**, No. 3, 53–59 (1995).
41. V. M. Teshukov, "Hydraulic jump in the shear flow of an ideal incompressible fluid," *Prikl. Mekh. Tekh. Fiz.*, **36**, No. 1, 11–20 (1995).
42. V. M. Teshukov, "Hydraulic jump in shear flow of a barotropic fluid," *Prikl. Mekh. Tekh. Fiz.*, **37**, No. 5, 73–81 (1996).
43. V. M. Teshukov, "Simple waves in a shear free-boundary flow of an ideal incompressible fluid," *Prikl. Mekh. Tekh. Fiz.*, **38**, No. 2, 48–57 (1997).
44. A. A. Chesnokov, "Exact solutions of the equations of eddying shallow water," *Prikl. Mekh. Tekh. Fiz.*, **38**, No. 5, 44–55 (1997).
45. B. N. Elemesova, "Simple waves in a layer of an ideal barotropic fluid," *Prikl. Mekh. Tekh. Fiz.*, **38**, No. 5, 56–64 (1997).
46. V. Yu. Liapidevskii, "A model of two-layer shallow water with an irregular interface," in: *Laboratory Modeling* (Collected scientific papers) [in Russian], Inst. of Thermal Phys., Novosibirsk (1991), pp. 87–97.
47. V. Yu. Liapidevskii, "Dynamics of a uniform turbulent layer in a stratified fluid," *Prikl. Mekh. Tekh. Fiz.*, **30**, No. 2, 73–76 (1989).
48. V. Yu. Liapidevskii, "Blocking of the flow of a two-layer immiscible fluid over an obstacle," *Prikl. Mat. Mekh.*, **58**, No. 4, 108–112 (1994).
49. V. Yu. Liapidevskii, "Mixing and blocking effects in two-layer flow over an obstacle," in: *Proc. 4th Int. Symp. on Stratified Flows*, **4** (1994), pp. 183–189.
50. V. Yu. Liapidevskii, "Forced two-layer flow regimes over an obstacle," *Dokl. Ross. Akad. Nauk*, **335**, No. 1, 61–63 (1994).
51. V. N. Liapidevskii, "Generation of long waves by the bottom contour in a two-layer flow," *Prikl. Mekh. Tekh. Fiz.*, **35**, No. 3, 34–37 (1994).

52. N. G. Gavrilov and V. Yu. Liapidevskii, "Anomalous two-layer flow regimes over an obstacle," *Prikl. Mekh. Tekh. Fiz.*, **37**, No. 4, 81–88 (1996).
53. I. V. Sturova, "Propagation of plane surface waves over a partially capped rectangular trench," *Prikl. Mekh. Tekh. Fiz.*, **32**, No. 5, 40–46 (1991).
54. I. V. Sturova, "Propagation of plane surface waves over an underwater rectangular step with overhang and over a submerged plate," *Prikl. Mekh. Tekh. Fiz.*, **32**, No. 3, 55–62 (1991).
55. A. A. Korobkin, "Propagation of internal waves in an exponentially stratified liquid above a protuberance," *Prikl. Mekh. Tekh. Fiz.*, **33**, No. 2, 32–38 (1992).
56. A. A. Korobkin, "Fundamental solution of a Cauchy–Poisson problem for a basin with an uneven bottom," *Prikl. Mekh. Tekh. Fiz.*, **31**, No. 2, 40–47 (1990).
57. A. A. Korobkin and I. V. Sturova, "Plane Cauchy–Poisson problem for a pond with a smoothly changing bottom. Examples of numerical computation," *Prikl. Mekh. Tekh. Fiz.*, **31**, No. 3, 54–60 (1990).
58. A. A. Korobkin and I. V. Sturova, "Cauchy–Poisson problem for a pond with a smoothly changing bottom," in: *Dynamics of Continuous Media* (Collected scientific papers) [in Russian], Novosibirsk, **97** (1990), pp. 37–50.
59. A. A. Korobkin and I. V. Sturova, "Numerical calculation of unsteady surface waves using Maslov's operator method," in: Proc. 1st Int. Conf. Math. and Numer. Aspects of Wave Propagation Phenomena, Strastburg, France (1991), pp. 577–591.
60. I. V. Sturova, "Scattering of internal waves generated by a moving body from periodic bottom roughness," *Izv. Akad. Nauk SSSR, Fiz. Atmos. Okeana*, **26**, No. 7, 755–762 (1990).
61. S. V. Sukhinin, "The waveguide effect," *Prikl. Mekh. Tekh. Fiz.*, **30**, No. 2, 92–101 (1989).
62. A. A. Korobkin, "Asymptotic analysis of the nonstationary problem of waves in a two-layer flow," *Izv. Akad. Nauk SSSR. Mekh. Zhidk. Gaza*, No. 6, 82–90 (1987).
63. V. I. Bukreev, A. V. Gusev, and I. V. Sturova, "Generation of internal waves under combined translational and vibrational motion of a cylinder in a fluid bilayer," *Prikl. Mekh. Tekh. Fiz.*, **27**, No. 3, 63–70 (1986).
64. V. I. Bukreev and N. V. Gavrilov, "Perturbations ahead of a wing moving in a stratified fluid," *Prikl. Mekh. Tekh. Fiz.*, **31**, No. 2, 102–105 (1990).
65. V. I. Bukreev, N. V. Gavrilov, and A. V. Gusev, "Internal waves in a pycnocline for a wing moving over a barrier," *Prikl. Mekh. Tekh. Fiz.*, **32**, No. 4, 68–74 (1991).
66. E. V. Ermanyuk, "Experimental study of the force of internal waves acting on a stationary sphere," *Prikl. Mekh. Tekh. Fiz.*, **34**, No. 4, 103–107 (1993).
67. N. A. Gavrilov and E. V. Ermanyuk, "Effect of pycnocline on forces exerted by internal waves on a stationary cylinder," *Prikl. Mekh. Tekh. Fiz.*, **37**, No. 6, 61–69 (1996).
68. V. I. Bukreev, A. V. Gusev, and E. V. Ermanyuk, "Experimental study of the motion of a submerged body on internal waves," *Izv. Ross. Akad. Nauk., Mekh. Zhidk. Gaza*, No. 2, 199–203 (1995).
69. N. V. Gavrilov, E. V. Ermanyuk, and I. V. Sturova, "Phenomenon of predominant orientation of a submerged elliptical cylinder under the action of surface waves," *Prikl. Mekh. Tekh. Fiz.*, **37**, No. 3, 25–34 (1996).
70. V. I. Bukreev, "Instability of internal waves from a cylinder in a shear flow with a large Richardson number," *Prikl. Mekh. Tekh. Fiz.*, **29**, No. 5, 85–89 (1988).
71. I. V. Sturova, "Effect of internal waves on the hydrodynamic characteristics of a submerged body," *Izv. Akad. Nauk SSSR, Fiz. Atmos. Okeana*, **26**, No. 6, 732–738 (1993).
72. I. V. Sturova, "Effect of anomalous dispersion dependences on scattering and generation of internal waves," *Prikl. Mekh. Tekh. Fiz.*, **35**, No. 3, 47–53 (1994).
73. I. V. Sturova, "Plane problem of hydrodynamic shaking of a submerged body without motion in a two-layered fluid," *Izv. Ross. Akad. Nauk, Mekh. Zhidk. Gaza*, No. 3, 144–155 (1994).
74. I. V. Sturova, "Plane problem of hydrodynamic shaking of a submerged body in the presence of motion in a two-layered fluid," *Prikl. Mekh. Tekh. Fiz.*, **35**, No. 5, 32–44 (1994).

75. T. I. Khabakhpasheva, "Plane problem of two-layer flow past a circular cylinder," *Izv. Ross. Akad. Nauk, Mekh. Zhidk. Gaza*, No. 1, 91-97 (1996).
76. T. I. Khabakhpasheva, "Diffraction of internal waves by a cylinder in a two-layer fluid," *Izv. Akad. Nauk SSSR, Fiz. Atmos. Okeana*, **29**, No. 4, 559–564 (1993).
77. N. V. Gavrilov, E. V. Ermanyuk, and I. V. Sturova, "The forces exerted by internal waves on a restrained body submerged in a stratified fluid," in: Proc. 21th Symp. on Naval Hydrodynamics, Trondheim, Norway (1996), pp. 254–265.
78. D. N. Gorelov and S. I. Gorlov, "Motion of an airfoil near a flat screen," *Prikl. Mekh. Tekh. Fiz.*, **36**, No. 1, 47–52 (1995).
79. D. N.Gorelov and S. I. Gorlov, "The linear problem of a hydrofoil moving under an interface between two heavy fluids," *Prikl. Mekh. Tekh. Fiz.*, **37**, No. 5, 43–47 (1996).
80. S. I. Gorlov, "Motion of a hydrofoil above an interface between two heavy fluids," *Prikl. Mekh. Tekh. Fiz.*, **37**, No. 5, 48–51 (1996).
81. S. I. Gorlov, "Solution of linear problems of uniform motion of a vorticity source in a multilayer fluid," *Izv. Ross. Akad. Nauk, Mekh. Zhidk. Gaza*, No. 3, 127–132 (1996).
82. S. I. Gorlov, "Effect of internal linear waves on the hydrodynamic characteristics of a vorticity source," *Izv. Ross. Akad. Nauk, Mekh. Zhidk. Gaza*, No. 5, 146–153 (1996).
83. L. A. Tkacheva, "Unsteady motion of a thin wing in a stratified fluid," *Prikl. Mekh. Tekh. Fiz.*, **36**, No. 6, 37–49 (1995).
84. A. M. Meirmanov, *The Stefan Problem* [in Russian], Nauka, Novosibirsk (1986).
85. V. N. Starovoitov, Solvability of the Stefan problem with the Gibbs–Tomson condition at the interface in small time," in: *Dynamics of Continuous Media* (Collected scientific papers) [in Russian], Novosibirsk, **95** (1990), pp. 151–156.
86. I. A. Kaliev, "Nonequilibrium phase transitions in frozen grounds," *Free Boundary Problems in Continuum Mechanics. Int. Ser. of Numerical Math.*, **106** (1992), pp. 141–148.
87. P. I. Plotnikov and V. N. Starovoitov, "The Stefan problem with surface tension as the limit of the equations of the phase field," *Differ. Uravn.*, **29**, No. 3, 461–470 (1993).
88. P. I. Plotnikov, "Equations with a variable direction of parabolicity and the hysteresis effect," *Dokl. Ross. Akad. Nauk*, **330**, No. 6, 691–693 (1993).
89. P. I. Plotnikov, "Passage to the limit for viscosity in equations with a variable direction of parabolicity," *Differ. Uravn.*, **30**, No. 3, 665–675 (1994).
90. P. I. Plotnikov, "Forward-backward parabolic equations and hysteresis," in: *Zap. Nauch. Seminara POMI*, **28**, No. 233, 191–216 (1996).
91. P. I. Plotnikov, "Passage to the limit for the small parameter in the Kan–Hillard equation," *Sib. Mat. Zh.*, **38**, No. 3 (1997).
92. V. N. Starovoitov, "Model of the motion of a two-component liquid with allowance for capillary forces," *Prikl. Mekh. Tekh. Fiz.*, **35**, No. 6, 61–71 (1994).
93. V. K. Andreev and O. V. Admaev, "Axisymmetric thermocapillary flow in cylinder and cylindrical layer," *Hydromech. and Heat / Mass Transfer in Microgravity*, Gordon Breach Sci. Publ., Amsterdam (1992), pp. 169–172.
94. V. K. Andreev and O. V. Admaev, "Modeling of crystal growth by crucible-free zone melting, in: *Problems of Ensuring the Quality of Articles in Machine Building, Proc. Int. Conf.* [in Russian], Izd. Krasnoyarsk Univ., Krasnoyarsk (1994), pp. 372–380.
95. V. K. Andreev, A. A. Rodionov, and E. A. Ryabitskii, "Development of thermocapillary convection in a fluid cylinder and cylindrical and plane layer under the influence of internal heat sources," *Prikl. Mekh. Tekh. Fiz.*, **30**, No. 2, 101–108 (1989).
96. V. K. Andreev and E. A. Ryabitskii, "Numerical study of thermocapillary instability in a cylindrical layer," in: *Hydrodynamics and Heat-and-Mass Exchange in Nongravity* [in Russian], Izd. Novosibirsk Univ., Novosibirsk (1988), pp. 35–44.

97. E. A. Ryabitskii, "Oscillatory thermocapillary instability of a liquid layer heated from below," in: *Microgravity Sci. Technol.*, VIII/3, Munich (1995), pp. 1–5.
98. V. K. Andreev and E. A. Ryabitskii, "Numerical investigation of thermocapillary instability of Marangoni convection in a cylindrical layer, in: *Microgravity Sci. Technol.*, VII/1, Munich (1994), pp. 36–40.
99. E. A. Ryabitskii, "Stability of thermocapillary motion in a cylindrical layer," *Prikl. Mekh. Tekh. Fiz.*, **30**, No. 4, 50–52 (1989).
100. L. G. Badratinova, M. Hennenberg, P. Colinet, and J. C. Legros, "On Rayleigh–Taylor instability in liated liquid-gas and liquid-vaper liate,"*Russ. J. Eng. Thermophys.*, **6**, No. 1, 1–31 (1996).
101. L. G. Badratinova, M. Hennenberg, P. Colinet, and J. C. Legros, "Theoretical models for boiling at microgravity," *Lecture Notes in Phys.*, **464**, 361–370 (1996).
102. L. G. Badratinova, "Crisis of transition of the bubble boiling regime to the sheet regime," in: *Dynamics of Continuous Media* (Collected scientific papers) [in Russian], Novosibirsk, **111** (1996), pp. 15–20.
103. V. V. Pukhnachov and O. V. Voinov, "Mathematical model of motion of emulsion under the effect of thermocapillary forces and microacceleration," in: Abstracts Ninth Europ. Symp. of Gravity Dependent Phenomena in Phys. Sci., Berlin (1995), pp. 32–33.
104. V. V. Pukhnachov and O. V. Voinov, "Thermocapillary motion in an emulsion," in: *Fluid Physics in Microgravity*, Proc. 3rd NASA Conf., Cleveland (1996), pp. 337–342.
105. E. A. Ryabitskii, "Oscillatory thermocapillary instability of the equilibrium of a flat layer in the presence of surfactants," *Izv. Ross. Akad. Nauk, Mekh. Zhidk. Gaza*, No. 1, 6–10 (1993).
106. E. A. Ryabitskii, "Thermocapillary instability of the equilibrium of a flat layer in the presence of surfactants," *Izv. Ross. Akad. Nauk, Mekh. Zhidk. Gaza*, No. 1, 3–8 (1996).
107. L. K. Antanovskii, "Influence of surface-active agents on the thinning of a vertical liquid film," *Prikl. Mekh. Tekh. Fiz.*, **35**, No. 2, 54–58 (1994).
108. V. V. Pukhnachov "Model for convective flow with decreased gravity," in: *Simulation in Mechanics* (Collected scientific papers) [in Russian], Comput. Center–Inst. of Theor. and Appl. Mech., Sib. Div., Acad. of Sci. of the USSR, **6(23)**, No. 4 (1992), pp. 47–58.
109. V. V. Pukhnachov, "Solvability of initial boundary-value problem in nonstandard model of convection," *Zap. Nauch. Seminara POMI*, **233**, 217–226 (1996).
110. V. V. Pukhnachov, "Steady problem of microconvection," in: *Dynamics of Continuous Media* (Collected scientific papers) [in Russian], Novosibirsk, **111** (1996), pp. 109–116.
111. V. V. Pukhnachov, "Microconvection in a vertical layer," *Izv. Ross. Akad. Nauk, Mekh. Zhidk. Gaza*, No. 5, 76–84 (1994).
112. O. N. Goncharova, "Microconvection in weak force fields. A numerical comparison of two models," *Prikl. Mekh. Tekh. Fiz.*, **38**, No. 2, 58–63 (1997).
113. O. N. Goncharova, "Numerical simulation of microconvection in domains with free boundaries," *Prikl. Mekh. Tekh. Fiz.*, **38**, No. 3, 64–68 (1997).
114. V. L. Sennitskii, "Motion of a circular cylinder in a vibrating liquid," *Prikl. Mekh. Tekh. Fiz.*, **26**, No. 5, 19–23 (1985).
115. B. A. Lugovtsov and V. L. Sennitskii, "Motion of a body in a vibrating liquid," *Dokl. Akad. Nauk SSSR*, **289**, No. 2, 314–317 (1986).
116. V. L. Sennitskii, "Motion of a gas bubble in a viscous vibrating liquid," *Prikl. Mekh. Tekh. Fiz.*, **29**, No. 6, 107–113 (1988).
117. V. L. Sennitskii, "Predominantly unidirectional motion of a gas bubble in a vibrating liquid," *Dokl. Akad. Nauk SSSR*, **319**, No. 1, 117–119 (1991).
118. V. L. Sennitskii, "Predominantly unidirectional motion of a compressible body in a vibrating liquid," *Prikl. Mekh. Tekh. Fiz.*, **34**, No. 1, 100–101 (1993).
119. V. L. Sennitskii, "Motion of a sphere in a fluid caused by vibrations of another sphere," *Prikl. Mekh. Tekh. Fiz.*, **27**, No. 4, 31–36 (1986).

120. V. L. Sennitskii, "On motion of inclusions in uniformly and nonuniformly vibrating liquid," in: *Proc. Int. Workshop on G-Jitter*, Clarkson Univ., Potsdam (1993).
121. O. M. Lavrent'eva, "On motion of a solid body in an ideal vibrating liquid," in: *Dynamics of Continuous Media* (Collected scientific papers) [in Russian], Novosibirsk, **103** (1991), pp. 120–125.
122. A. A. Kashevarov, "Numerical simulation of the interrelation between pressure filtration and surface drainage," *Dynamics of Continuous Media* (Collected scientific papers) [in Russian], Novosibirsk, **111**, (1996), pp. 40–48.
123. S. N. Antontsev and A. A. Kashevarov, "Localization of solutions of nonlinear parabolic equations degenerating on the surface, *ibid.*, pp. 7–20.
124. V. N. Emikh, *Hydrodynamics of Filtration Flows with Drainage* [in Russian], Nauka, Novosibirsk (1993).
125. V. I. Pen'kovskii and S. T. Rybakova, "Crust formation in borehole drilling," *Dynamics of Continuous Media* (Collected scientific papers) [in Russian], Novosibirsk, **90**, 72–80 (1989).
126. V. I. Pen'kovskii and S. T. Rybakova, "Numerical simulation of heat-and-mass transfer for underground leaching," *ibid.*, pp. 81–92.
127. Yu. I. Kapranov, "Structural model of mechanical mud injection into a porous medium," *ibid.*, pp. 27–39.
128. A. V. Domanskii, "Stabilization of the generalized solutions of degenerate problems of two-phase filtration," *Differ. Uravn.*, **27**, No. 4, 641–648 (1991).
129. S. N. Antontsev, N. I. Diaz, and A. V. Domanskii, "Stability and stabilization of the generalized solutions of degenerate problems of two-phase filtration," *Dokl. Ross. Akad. Nauk*, **326**, No. 6, 1151–1155 (1992).
130. S. N. Antontsev, A. V. Domanskii, and V. I. Pen'kovskii, *Filtartion in the Borehole Zone of a Bed and Intensification of Inflow* [in Russian], Institute of Hydrodynamics, Novosibirsk (1989).
131. V. I. Pen'kovskii, "Capillary pressure and the gravity and dynamic phase distribution in a water-oil-gas-rock system," *Prikl. Mekh. Tekh. Fiz.*, **37**, No. 6, 85–90 (1996).
132. V. N. Monakhov and N. V. Khusnutdinova, "Conjugation of channel and filtration flows of a viscous incompressible fluid," *Prikl. Mekh. Tekh. Fiz.*, **36**, No. 1, 95–99 (1995).
133. V. A. Vladimirov, "Instability of the equilibrium of fluids," *Prikl. Mekh. Tekh. Fiz.*, **30**, No. 2, 108–116 (1989).
134. V. A. Vladimirov, "Variational approaches in the theory of stability of the baroclinic atmosphere," *Izv. Akad. Nauk SSSR, Fiz. Atmos. Okeana*, **25**, No. 4, 348–355 (1995).
135. V. A. Vladimirov, "Instability of the equilibrium of a viscous capillary fluid," in: *Dynamics of Continuous Media* (Collected scientific papers) [in Russian], Novosibirsk, **89** (1989), 92–106.
136. V. A. Vladimirov and V. V. Rumyantsev, "On the generalization of the Lagrange theorem for a solid body with a cavity containing an ideal fluid," *Prikl. Mat. Mekh.*, **53**, No. 2, 608–612 (1989).
137. V. A. Vladimirov and V. V. Rumyantsev, "On the generalization of the Lagrange theorem for a solid body with a cavity containing a viscous fluid," *Prikl. Mat. Mekh.*, **54**, No. 2, 190–200 (1990).
138. K. I. Il'in, "Instability of the states of equilibrium for liquid crystals," in: *Dynamics of Continuous Media* (Collected scientific papers) [in Russian], Novosibirsk, **96** (1990), 111–121.
139. Yu. G. Gubarev, "On the inversion of the Lagrange theorem in magnetohydrodynamics," *Prikl. Mat. Mekh.*, **54**, No. 6, 988–991 (1990).
140. Yu. G. Gubarev, "Instability of a self-gravitating compressible medium," *Prikl. Mekh. Tekh. Fiz.*, **35**, No. 4, 68–77 (1994).
141. K. I. Il'in, "On the stability of a baroclinic vortex," *Izv. Akad. Nauk SSSR, Fiz. Atmos. Okeana*, **27**, No. 5, 584–588 (1991).
142. Yu. G. Gubarev, "On the instability of oscillatory symmetrical MHD flows," *Izv. Ross. Akad. Nauk, Mekh. Zhidk. Gaza*, No. 6, 19–25 (1995).
143. V. A. Vladimirov, "Analogs of the Lagrange theorem in the hydrodynamics of eddying and stratified fluids," *Prikl. Mat. Mekh.*, **50**, No. 5, 727–733 (1986).

144. V. A. Vladimirov, "Conditions for nonlinear stability of flows of an ideal incompressible liquid," *Prikl. Mekh. Tekh. Fiz.*, **27**, No. 3, 70–78 (1986).
145. V. A. Vladimirov, "On nonlinear stability of incompressible fluid flows," *Arch. Mech.*, **38**, Nos. 5 and 6, 689–696 (1986).
146. V. L. Vladimirov, "Stability of flows with vortex discontinuity," *Prikl. Mekh. Tekh. Fiz.*, **29**, No. 1, 83–91 (1988).
147. V. L. Vladimirov and Yu. G. Gubarev, "Conditions for nonlinear stability of plane and spiral MHD flows," *Prikl. Mekh. Tekh. Fiz.*, **59**, No. 3, 442–450 (1995).
148. S. Ya. Belov and V. A. Vladimirov, "An example of inversion of the Lagrange theorem in the hydrodynamics of a two-layer fluid," *Dynamics of Continuous Media* (Collected scientific papers) [in Russian], Novosibirsk, **84** (1988), pp. 21–27.
149. M. A. Gol'dshtik, E. M. Zhdanova, and V. N. Shtern, "Spontaneous swirling of a buoyant jet," *Dokl. Akad. Nauk SSSR*, **277**, No. 4, 815–818 (1984).
150. M. A. Gol'dshtik, V. N. Shtern, and N. I. Yavorskii, *Viscous Flows with Paradoxical Properties* [in Russian], Nauka, Novosibirsk (1989).
151. M. A. Gol'dshtik and V. N. Shtern, "Turbulent vortex dynamo," *Prikl. Mat. Mekh.*, **53**, No. 4, 613–624 (1989).
152. B. A. Lugovtsov, "Is spontaneous swirling of axisymmetric flow possible?" *Prikl. Mekh. Tekh. Fiz.*, **35**, No. 2, 50–54 (1994).
153. Yu. G. Gubarev and B. A. Lugovtsov, "On spontaneous swirling in axisymmetric flows," *Prikl. Mekh. Tekh. Fiz.*, **36**, No. 4, 52–59 (1995).
154. B. A. Lugovtsov, "Spontaneous swirling in axisymmetric flows of a conducting fluid in a magnetic field," *Prikl. Mekh. Tekh. Fiz.*, **37**, No. 6, 36–43 (1996).
155. B. A. Lugovtsov, "Axisymmetric spontaneous swirling in an ideal conducting fluid in a magnetic field," *Prikl. Mekh. Tekh. Fiz.*, **38**, No. 6, 29–31 (1997).
156. O. V. Kaptsov, "Some classes of two-dimensional vortex flows of an ideal fluid," *Prikl. Mekh. Tekh. Fiz.*, **30**, No. 1, 109–117 (1989).
157. O. V. Kaptsov, "Steady vortex structures in an ideal fluid," *Zh. Éksp. Teor. Fiz.*, No. 1, 109–117 (1989).
158. R. M. Garipov, "An ellipsoidal drop or vortex in an inhomogeneous flow," *Prikl. Mekh. Tekh. Fiz.*, **37**, No. 4, 76–80 (1996).
159. A. F. Voevodin and A. S. Ovcharova, "On the calculation of a vortex fucntion at the boundary of a closed circular region," *Model. Mekh.*, **5(22)**, No. 1, 115–120 (1991).
160. A. F. Voevodin, "Stability and realization of implicit schemes for the Stokes equations," *Zh. Vychisl. Mat. Mat. Fiz.*, **33**, No. 1, 119–130 (1993).
161. A. F. Voevodin, V. V. Ostapenko, Yu. V. Pibobarov, and S. M. Shugrin, *Problems of Computational Mathematics* [in Russian], Izd. Sib. Otdel. Ross. Akad. Nauk, Novosibirsk (1995).
162. A. F. Voevodin, "Stability of the difference boundary-value problem for the Stokes equations in a doubly-connected circular region," in: *Vychisl. Tekhnol.*, **4**, No. 12, 70–76 (1995).
163. V. V. Kuznetsov, "On the conditions of solvability of the boundary-value problem for the Prandtl boundary layer," *Sib. Mat. Zh.*, **35**, No. 3, 624–629 (1994).
164. D. G. Akhmetov and V. F. Tarasov, "Structure and evolution of vortex cores," *Prikl. Mekh. Tekh. Fiz.*, **27**, No. 5, 68–73 (1986).
165. V. G. Makarenko and V. S. Tarasov, "Flow structure of a rotating liquid after motion of a body in it," *Prikl. Mekh. Tekh. Fiz.*, **29**, No. 6, 113–117 (1988).
166. V. G. Makarenko and V. F. Tarasov, "Experimental model of the tornado," *Prikl. Mekh. Tekh. Fiz.*, **28**, No. 5, 115–122 (1987).
167. G. Makarenko and V. F. Tarasov, "Experimental model of the tornado," *Dokl. Akad. Nauk SSSR*, **305**, No. 2, 297–300 (1989).

168. V. G. Makarenko, "The characteristics of inertial resonances in a rotating liquid," *Prikl. Mekh. Tekh. Fiz.*, **34**, No. 2, 89–96 (1993).
169. V. G. Makarenko, "Dependence of formation time of oscillating vortices on excitation frequency of a rotating fluid," *Prikl. Mekh. Tekh. Fiz.*, **36**, No. 1, 85–94 (1995).
170. V. V. Nikulin, "Conical vortex flow induced by tangential stresses on a free plane surface," *Prikl. Mat. Mekh.*, **52**, No. 4, 594–600 (1988).
171. V. V. Nikulin, "Space evolution of tornado-like vortex core," *Int. Ser. Numer. Math.*, **106**, Birkhauser Verlag, Basel (1992), pp. 229–237.
172. V. V. Nikulin, "Analog of the shallow-water vortex equation for hollow and tornado-like vortices. Height of a steady tornado-like vortex," *Prikl. Mekh. Tekh. Fiz.*, **33**, No. 2, 47–52 (1992).
173. V. V. Nikulin, "Decay of a vertical tornado-like vortex," *Prikl. Mekh. Tekh. Fiz.*, **33**, No. 4, 42–47 (1992).
174. V. V. Nikulin, "Motion of the swirled fluid in the core of a vertical, tornado-like vortex," *Prikl. Mekh. Tekh. Fiz.*, **36**, No. 2, 81–87 (1995).
175. V. V. Nikulin, "A hollow vortex with an axial velocity in a tube of variable radius," *Prikl. Mat. Mekh.*, **58**, No. 2, 49–53 (1994).
176. B. A. Lugovtsov, "Determination of the main flow parameters in a swirl atomizer by means of conservation laws," *Prikl. Mekh. Tekh. Fiz.*, **30**, No. 2, 68–73 (1989).
177. B. A. Lugovtsev, "Principle of maximum discharge," *Prikl. Mekh. Tekh. Fiz.*, **32**, No. 4, 105–106 (1991).
178. V. I. Bukreev, "Experimental study of the range of application of the solution of the Stokes second problem," *Izv. Ross. Akad. Nauk, Mekh. Zhidk. Gaza*, No. 4, 26–31 (1988).
179. N. V. Alekseenko and V. A. Kotomakha, "Experimental study of an axisymmetric momentum-free turbulent jet," *Prikl. Mekh. Tekh. Fiz.*, **28**, No. 1, 65–69 (1987).
180. V. A. Kostomaha and N. V. Lesnova, "Turbulent swirling wake behind a sphere with complete or partial drag compensation," *Prikl. Mekh. Tekh. Fiz.*, **36**, No. 2, 88–98 (1995).
181. V. I. Bukreev, A. G. Demenkov, V. A. Kostomakha, and G. G. Chernykh, "Heat propagation from a linear source in a plane turbulent wake," *Prikl. Mekh. Tekh. Fiz.*, **37**, No. 5, 115–126 (1996).
182. V. P. Ryabchenko, "Calculation of the unsteady aerodynamic characteristics of an annular blade cascade of arbitrary shape," *Tr. TsIAM*, No. 1221, 4–14 (1987).
183. V. P. Ryabchenko, "Unsteady aerodynamic characteristics of a three-dimensional blade cascade in a subsonic gas flow," *Prikl. Mekh. Tekh. Fiz.*, **36**, No. 2, 45–55 (1995).
184. V. B. Kurzin, "Flow of an ideal fluid induced by a periodic system of vortex wakes," in: *Dynamics of Continuous Media* (Collected scientific papers) [in Russian], Novosibirsk, **87**, 88–96 (1987).
185. V. A. Yudin, "Airfoil cascade in unsteady eddy flow," *Prikl. Mekh. Tekh. Fiz.*, **35**, No. 4, 85–91 (1994).
186. R. A. Izmailov, V. B. Kurzin, and V. L. Okulov, "Acoustic resonance in subsonic aerodynamic interaction of cascades," *Prikl. Mekh. Tekh. Fiz.*, **28**, No. 1, 20–27 (1987).
187. V. B. Kurzin and O. B. Ovsyannikova, "On the stability of a vortex source at the center of a rotating annular airfoil cascade," *Tr. TsIAM*, No. 1293, 87–98 (1991).
188. V. B. Kurzin, S. N. Korobeinikov, V. P. Ryabchenko, and L. A. Tkacheva, "Natural vibrations of a uniform cascade of hydraulic-turbine blades in a fluid," *Prikl. Mekh. Tekh. Fiz.*, **38**, No. 2, 80–90 (1997).
189. V. B. Kurzin, "Asymptotic model of an active resonance absorber of acoustic vibration in a bounded region," *Prikl. Mekh. Tekh. Fiz.*, **35**, No. 6, 44–55 (1994).
190. V. B. Kurzin, "Hydroelastic oscillations of the active portion of an inductive source of energy," *Izv. Ross. Akad. Nauk, Mekh. Tverd. Tela*, No. 2, 151–158 (1994).
191. S. V. Sukhinin and V. F. Akhmadev, "Hydrodynamic sources of oscillations in combustion chambers," *Fiz. Goreniya Vzryva*, **29**, No. 6, 38–46 (1993).

192. S. V. Sukhinin and V. F. Akhmadev, "Oscillations and vortices in a combustion chamber," *Izv. Vyssh. Uchebn. Zaved., Fiz.*, No. 4, 111–118 (1994)
193. A. A. Korobkin and V. V. Pukhnachov, "Initial stage of water impact," *Ann. Rev. Fluid Mech.*, **20**, 159–185 (1988).
194. A. A. Korobkin, "Acoustic effects on water impact," Proc. 10th Int. Workshop on Water Waves and Floating Bodies, Oxford (1995), pp. 25.1–25.4.
195. A. A. Korobkin, "Acoustic approximation in the penetration of an ideal fluid by a blunted contour," *Prikl. Mekh. Tekh. Fiz.*, **33**, No. 4, 48–54 (1992).
196. A. A. Korobkin, "Blunt-body impact on the free surface of a compressible liquid," *J. Fluid Mech.*, **263**, 319–342 (1994).
197. A. A. Korobkin, "Impact on the surface of a compressible liquid," in: *Propagation of a Weak, Nearly Plane Shock Wave. Shock Waves*, **4**, No. 4, 209–216 (1995).
198. A. A. Korobkin, "Impact on the surface of a compressible liquid," in: *Focusing of a Weak, Near-Cylindrical Shock Wave. Shock Waves*, **4**, No. 4, 217–224 (1995).
199. A. A. Korobkin, "Jetting under body impact on a liquid free surface," in: Proc. 4th Int. Symp. on Nonlinear and Free-Surface Flows, Hiroshima Univ. (1995), pp. 5–8.
200. A. A. Korobkin, "Low-pressure zones under a liquid–solid impact," *Fluid Mech. Its Appl.*, **23**, 375–381 (1994).
201. A. A. Korobkin, "Acoustic approximation in the slamming problem," *J. Fluid Mech.*, **318**, 165–188 (1996).